B.Sc Part II (Hons).
Paper IV

Formation and solution of differential equation.

Definition:_ An equation involving the dependent variable and independent variable and also the derivatives of the dependable variable is known as a differential equation.

Example - Let $y = mx + m^2$ ——— (1)

be an equation containing x and y and one arbitrary constant m.

Differentiating with respect to x, we get

$$\frac{dy}{dx} = m \quad \text{——— (2)}$$

Now eliminating m between (1) and (2), we have

$$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2$$

Which gives us a diffential equation involving x, y and $\frac{dy}{dx}$.

Order:_ The order of a differential equation is the order of the highest derivative (differential coefficient) involved in its expression.

$\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 4x = 0$ is the differential equation of the first order.

Here the maximum derivative of y w.r.t x is $\frac{dy}{dx}$.

Similarly each of the equations

$$\frac{d^2y}{dx^2} + 4y = e^x, \quad \left\{1 + \left(\frac{dy}{dx}\right)\right\}^{3/2} = k\frac{d^2y}{dx^2}$$

is of second order.

Degree:- The degree of a differential equation can be defined as the degree of the highest order of differential coefficient when the equation has been made rational (free from any radicals) and integral as far as the differential coefficients are concerned.

Thus $\left(\frac{dy}{dx}\right)^2 - 5\frac{dy}{dx} + 6y = 0$

(though of first order) is of second degree.

Now consider the differential equation

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = k\frac{d^2y}{dx^2}$$

squaring it (so that it may be rationalised), we get

$$\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2.$$

Since $\frac{dy^2}{dx^2}$ occurs squared, we find that the given differential equation is of second degree.

Ex- from the differential equation for the curve

$$y^2 = 4a(x+a).$$

Solution:- From the given equation $y^2 = 4a(x+a)$, we have

$$y^2 = 4ax + 4a^2 \quad \text{——— (1)}$$

differentiating, we get

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$$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a \quad (2)$$

Hence eliminating (a) from (1) and (2), we get

$$y^2 = 4x\frac{1}{2}y\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$= 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$= y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

Which is the required differential equation.

Ex:- Find the differential equation corresponding to

$$y = ae^{2x} + be^{-3x} + ce^{x}$$

Where a, b, c are arbitrary constants.

Solution:- Since the given function involves three constants, therefore the differential equation will involve a differential coefficient up to third order.

Given $y = ae^{2x} + be^{-3x} + ce^{x}$

therefore $y_1 = \frac{dy}{dx} = 2ae^{2x} - 3be^{-3x} + ce^{x}$

$$y_2 = \frac{d^2y}{dx^2} = 4ae^{2x} + 9be^{-3x} + ce^{x}$$

$$y_3 = \frac{d^3y}{dx^2} = 8ae^{2x} - 27be^{-3x} + ce^{x}.$$

Eliminating a, b, c from these equations, we get

$$\begin{vmatrix} y & e^{2x} & e^{-3x} & e^{x} \\ y_1 & 2e^{2x} & -3e^{-3x} & e^{x} \\ y_2 & 4e^{2x} & 9e^{-3x} & e^{x} \\ y_3 & 8e^{2x} & -27e^{-3x} & e^{x} \end{vmatrix} = 0$$

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$$\Rightarrow \begin{vmatrix} y & 1 & 1 & 1 \\ y_1 & 2 & -3 & 1 \\ y_2 & 4 & 9 & 1 \\ y_3 & 8 & -27 & 1 \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, $R_3 \to R_3 - R_4$, we get

$$\begin{vmatrix} y-y_1 & -1 & 4 & 0 \\ y_1-y_2 & -2 & -12 & 0 \\ y_2-y_3 & -4 & 36 & 0 \\ y_3 & 8 & -27 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} y-y_1 & -1 & 4 \\ y_1-y_2 & -2 & -12 \\ y_2-y_3 & -4 & 36 \end{vmatrix} = 0$$

Expanding, we get $y_3 - 7y_1 + 6 = 0$

i.e. $\dfrac{d^3y}{dx^3} - 7\dfrac{dy}{dx} + 6y = 0$

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